MENIIT
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JEE MAINS-2013

23-04-2013 (Online-3)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-PHYSICS

1. A particle of mass 2 kg is moving such that at time t, its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at t = 2s about the origin in kg m^{-2} s :

(1) $(10\hat{i} - 16\hat{j})$ (2) $-40\hat{k}$ (3) $40\hat{k}$ (4*) $-80\hat{k}$

Sol. Angular momentum L = m (v × r)

$$= 2kg\left(\frac{dr}{dt} \times r\right) = 2kg\left(4tj \times 5i - 2t^2\hat{j}\right)$$

$$= 2kg\left(-20t\hat{k}\right) = 2kg \times -20 \times 2m^{-2}s^{-1}\hat{k}$$

$$= -80\hat{k}$$

2. Consider a finite insulated, uncharged conductor placed near a finite positively charged conductor. The uncharged body must have a potential :

(A) more than the charged conductor and more than at infinity.

(B) less than the charged conductor and less than at infinity.

(C) more than the charged conductor and less than at infinity.

(D*) less than the charged conductor and more than at infinity.

Sol. The potential of uncharged body is less than that of the charged conductor and more than at infinity.

3. Wax is coated on the inner wall of a capillary tube and the tube is then dipped in water. Then, compared to the unwaxed capillary, the angle of contact θ and the height h upto which water rises change. These changes are :

(1) θ decreases and h also decreases (2) θ decreases and h increases

(3) θ increases and h also increases (4*) θ increases and h decreases

Sol. Angle of contact θ

$$\cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$$

when water is on a waxy or oily surface

$T_{SA} < T_{SL}$ $\cos\theta$ is negative i.e.,

$90^\circ < \theta < 0180^\circ$

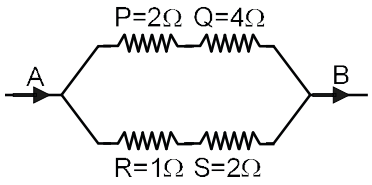
i.e., angle of contact θ increases

And for $\theta > 90^\circ$ liquid level in capillary tube fall i.e., h decreases

4. A body of mass 'm' is tied to one end of a spring and whirled round in a horizontal plane with a constant angular velocity. The elongation in the spring is 1 cm. If the angular velocity is doubled, the elongation in the spring is 5 cm. The original length of the spring is :

(1) 12 cm (2*) 15 cm (3) 10 cm (4) 16 cm

5. Which of the four resistances P, Q, R and S generate the greatest amount of heat when a current flows A to B ?

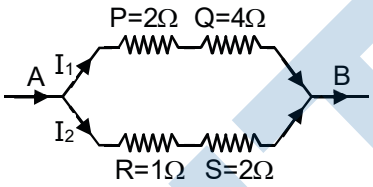


(1) P (2) Q (3) R (4*) S

Sol. $R_1 = P + Q = 2\ \Omega + 4\ \Omega = 6\ \Omega$

$R_2 = R + S = 1\ \Omega + 2\ \Omega = 3\ \Omega$

$I_1R_1 = I_2R_2$



$$I_1 = \frac{R_2}{R_1}I_2 = \frac{3}{6}I_2 = \frac{I_2}{2}$$

or $I_2 = 2I_1$

Heat flow $H = I^2 Rt$

For Q, $H_Q = I_1^2 Qt = \frac{I_2^2}{4} \times 4t = I_2^2 t$

For S, $H_S = I_2^2 St = I_2^2 \cdot 2t = 2I_2^2 t$

6. Electrons are accelerated through a potential difference V and protons are accelerated through a potential difference 4 V. The de-Broglie wavelengths are λ_e and λ_p for electrons and protons respectively.

The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by : (given m_e is mass of electron and m_p is mass of proton).

(1) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$ (2) $\frac{\lambda_e}{\lambda_p} = \frac{1}{2}\sqrt{\frac{m_e}{m_p}}$ (3) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_e}{m_p}}$ (4*) $\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$

Sol. Energy in joule (E)

= charge × potential diff. in volt

$E_{electron} = q_e V$ and $E_{proton} = q_p 4V$

de-Broglie wavelength $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mE}}$

$\lambda_e = \frac{h}{\sqrt{2m_e eV}}$ and $\lambda_P = \frac{h}{\sqrt{2m_P e4V}}$ ($\because q_e = q_P$)

$$\therefore \frac{\lambda_e}{\lambda_P} = \frac{\frac{h}{\sqrt{2m_e eV}}}{\frac{h}{\sqrt{2m_P e4V}}} = \sqrt{\frac{2m_P e4V}{2m_e eV}}$$

$$= 2\sqrt{\frac{m_P}{m_e}}$$

7. A liquid drop having 6 excess electrons is kept stationary under a uniform electric field of 25.5 kV m^{-1}. The density of liquid is 1.26×10^3 kg m^{-3}. The radius of the drop is (neglect buoyancy) :

(1*) 7.8×10^{-7} m (2) 4.3×10^{-7} m (3) 0.078×10^{-7} m (4) 3.4×10^{-7} m

Sol. $F = qE = mg$ ($q = 6e = 6 \times 1.6 \times 10^{-19}$)

$$\text{Density (d)} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi r^3}$$

$$\text{or } r^3 = \frac{m}{\frac{4}{3}\pi d}$$

Putting the value of d and $m\left(=\frac{qE}{g}\right)$ and solving we get $r = 7.8 \times 10^{-7}$ m

8. A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself. The centre of gravity rises by 0.5 m before he leaps. After the leap the c.g. rises by another 1 m. The maximum power delivered by the muscles is :

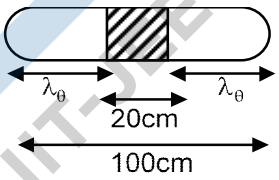
(Take g = 10 ms^{-2}) :

(1*) 6.26×10^3 Watts at take off (2) 6.26×10^4 Watts at take off

(3) 6.26×10^3 Watts at the start (4) 6.26×10^4 Watts at the start

9. A thin tube sealed at both ends is 100 cm long. It lies horizontally, the middle 20 cm containing mercury and two equal ends containing air at standard atmospheric pressure. If the tube is now turned to a vertical position, by what amount will the mercury be displaced ?

(Given : cross section of the tube can be assumed to be uniform)



(1) 0.0 cm (2) 2.95 cm (3) 8.65 cm (4*) 5.18 cm

10. 500 g of water and 100 g of ice at 0°C are in a calorimeter whose water equivalent is 40 g. 10 g of steam at 100°C is added to it. Then water in the calorimeter is :

(Latent heat of ice = 80 cal/g, Latent heat of steam = 540 cal/g) ?

(1) 610 g (2) 600 g (3) 580 g (4*) 590 g

Sol. As 1 g of steam at 100ºC melts 8 g of ice at 0ºC.

10 g of steam will melt 8 × 10 g of ice at 0ºC

Water in calorimeter = 500 + 80 + 10 g = 590 g

11. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ?

(1*) At 72 cm and 96 cm from one end (2) At 48 cm and 96 cm from one end

(3) At 24 cm and 72 cm from one end (4) At 36 cm and 84 from one end

Sol. Total length of the wire, L = 114 cm

$n_1 : n_2 : n_3 = 1 : 3 : 4$

Let L_1, L_2 and L_3 be the length of the three parts

As $n \propto \frac{1}{L}$

$\therefore \quad L_1 : L_2 : L_3 = \frac{1}{1} : \frac{1}{3} : \frac{1}{4} = 12 : 4 : 3$

$\therefore \quad L_1 = 72\text{cm}\left(\frac{12}{12+43+3} \times 114\right)$

$L_2 = 24\text{cm}\left(\frac{4}{19} \times 114\right)$

and $L_3 = 18\text{cm}\left(\frac{3}{19} \times 114\right)$

Hence the bridges should be placed at 72 cm and 72 + 24 = 96 cm from one end.

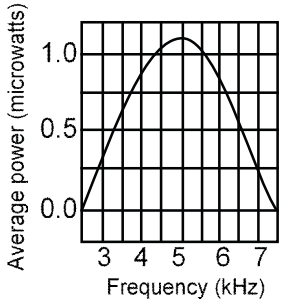
12. If the time period t of the oscillation of a drop of liquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{r^{2b}\, s^{c}\, d^{a/2}}$. It is observed that the time period is directly proportional to $\sqrt{\frac{d}{s}}$. The value of b should therefore be:

(A*) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $\sqrt{3}$ (D) $\frac{2}{3}$

13. A projectile of mass M is fired so that the horizontal range is 4 km. At the highest point the projectile explodes in two parts of masses M/4 and 3M/4 respectively and the heavier part starts falling down vertically with zero initial speed. The horizontal range (distance from point of firing) of the lighter part is :

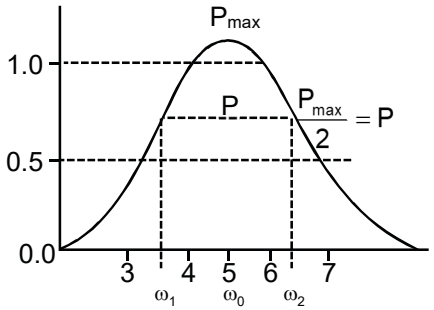
(1*) 10 km (2) 16 km (3) 1 km (4) 2 km

14. The plot given below is of the average power delivered to an LRC circuit versus frequency. The quality factor of the circuit is :



(1*) 2.0 (2) 0.4 (3) 2.5 (4) 5.0

Sol.



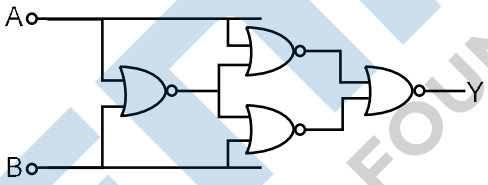
Quality factor of the circuit

$$= \frac{\omega_0}{\omega_2 - \omega_1} = \frac{5}{2.5} = 2.0$$

15. In the Bohr's model of hydrogen-like atom the force between the nucleus and the electron is modified as $F = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$, where β is a constant. For this atom, the radius of the n^{th} orbit in terms of the Bohr radius $\left(a_0 = \frac{\varepsilon_0 h^2}{m\pi e^2}\right)$:

(A) $r_n = a_0\, n - \beta$ (B) $r_n = a_0\, n^2 + \beta$ (C*) $r_n = a_0\, n^2 - \beta$ (D) $r_n = a_0\, n^2 + \beta$

16. A system of four gates is set up as shown. The 'truth table' corresponding to this system is :



(1*)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

(3)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(4)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

Sol. In the given system all four gate is NOR gate

dafddf

A	B	$(y' = \overline{A + B})$	$y'' = (\overline{A + y'})$	$y''' = (\overline{B + y'})$	$y = \overline{y'' + y'''}$
0	0	1	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

i.e.,

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

17. A parallel plate capacitor of area 60 cm^2 and separation 3 mm is charged initially to 90 μC. If the medium between the plate gets slightly conducting and the plate loses the charge initially at the rate of 2.5×10^{-8}C/s, then what is the magnetic field between the plates ?

(1) 1.63×10^{-11} T (2*) Zero (3) 2.0×10^{-7} T (4) 2.5×10^{-8}T

Sol. Magnetic field between the plates in this case is zero.

18. n identical waves each of intensity I_0 interfere with each other. The ratio of maximum intensities if the interference is (i) coherent and (ii) incoherent is :

(1) n^2 (2) $\frac{1}{n}$ (3*) n (4) $\frac{1}{n^2}$

Sol. $$\frac{\text{(Maximum intensity) coherent interference}}{\text{(Maximum intensity) coherent interference}}$$

$$= \frac{n^2 I_0}{nI_0} = n$$

19. Bob of a simple pendulum of length ℓ is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T then :

(1) $T < 2\pi\sqrt{\frac{\ell}{g}}$ and damping is smaller than in air alone

(2) $T > 2\pi\sqrt{\frac{\ell}{g}}$ and damping is smaller than in air alone

(3*) $T < 2\pi\sqrt{\frac{\ell}{g}}$ and damping is larger than in air alone

(4) $T = 2\pi\sqrt{\frac{\ell}{g}}$ and damping is larger than in air alone

Sol. When the pendulum is oscillating over a current carrying coil, and when the direction of oscillating pendulum bob is opposite to the direction of current. Its instantaneous acceleration increases.

Hence time period $T < 2\pi\sqrt{\frac{\ell}{g}}$

and damping is larger than in air alone due energy dissipation.

20. This question has statement-1 and statement-2. Of the four choice given after the statements, choose the one that best describes the two statements.

Statement1 : The internal energy of a perfect gas is entirely kinetic and depends only on absolute temperature of the gas at constant pressure is lower than that at constant volume:

Statement2 : A perfect gas is heated keeping pressure constant and later a constant volume. For the same amount of heat the temperature of the gas at constant pressure is lower than that at constant volume.

(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1

(2) Statement-1 is false, statement-2 is true

(3*) Statement-1 is false, statement-2 true but statement-2 is not the correct explanation of statement-1

(4) Statement-1 is true, statement-2 is false

Sol. Internal energy of a system is the energy possessed by the system due to molecular motion by the system due to molecular motion and molecular configuration.

Internal energy $u = u_k + u_p$

For an ideal gas there is no molecular attraction $u_p = 0$

$\therefore$ Internal energy $u = u_k$

(i.e., internal kinetic energy)

and $u = u_k = \frac{3}{2}\mu RT$

At constant pressure $(\Delta Q)_P = \mu C_P \Delta T$

At constant volume $(\Delta Q)_V = \mu C_V \Delta T$

As $C_P > C_V$ $\therefore$ $(\Delta T)_P < (\Delta T)_V$

21. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional area are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the young's modulii of copper and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is :

(1) 1.75 mm (2*) 1.25 mm (3) 2.0 mm (4) 1.50 mm

Sol. $Y_C \times (\Delta L_c / L_c) = Y_s \times (\Delta L_s / L_s)$

$$\Rightarrow 1\times 10^{11} \times \left(\frac{1\times 10^{-3}}{1}\right) = 2\times 10^{11} \times \left(\frac{\Delta L_s}{0.5}\right)$$

$$\therefore \Delta L_s = \frac{0.5\times 10^{-3}}{2} = 0.25\text{mm}$$

Therefore, total extension of the composite wire = $\Delta L_c + \Delta L_s$

= 1 mm + 0.25 m = 1.25 m

22. Select the correct statement from the following:

(1) electromagnetic waves are longitudinal waves

(2) Electromagnetic waves cannot travel in vacuum

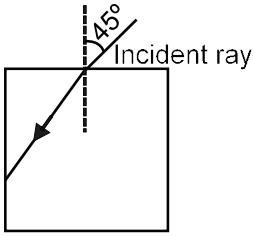
(3) Electromagnetic waves are produced by charges moving with uniform velocity

(4*) Electromagnetic waves carry both energy and momentum as they propagate through space.

Sol. Electromagnetic waves do not required any medium to propagate. They can travel in vacuum. They are transverse in nature like light. They carry both energy and momentum.

A changing electric field produces a changing magnetic field and vice-versa. Which gives rise to a transverse wave known as electromagnetic wave.

23. A light ray falls on a square glass slab as shown in the diagram. The index of refraction of the glass, if total internal reflection is to occur at the vertical face, is equal to :

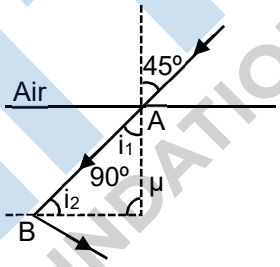


(1) $\frac{3}{2}$ (2*) $\sqrt{\frac{3}{2}}$ (3) $(\sqrt{2}+1)/2$ (4) $\sqrt{\frac{5}{2}}$

Sol. At point A by Snell's law

$$\mu = \frac{\sin 45^\circ}{\sin r} \Rightarrow \sin r = \frac{1}{\mu\sqrt{2}} \quad \text{....(i)}$$

At point B, for total internal reflection,



$$\sin i_1 = \frac{1}{\mu}$$

From figure, $i_1 = 90^\circ - r$

$$\therefore (\sin 90^\circ - r) = \frac{1}{\mu}$$

$$\Rightarrow \cos r = \frac{1}{\mu} \quad \text{....(ii)}$$

$$\text{Now } \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{1}{2\mu^2}}$$

$$= \sqrt{\frac{2\mu^2-1}{2\mu^2}}$$

From eqs. (ii) and (iii)

Squaring both sides and then solving, we get

$$\mu = \sqrt{\frac{3}{2}}$$

24. Which of the following statement is NOT correct ?

(1) Sky wave signals in the broadcast frequency range are stronger at night than in the day time

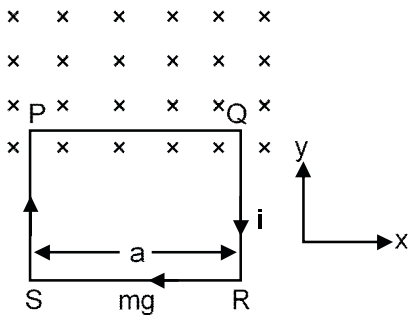
(2) The critical frequency of an ionosphere layer is the highest frequency that will be reflected by the layer when it is vertically incident

(3*) Electromagnetic waves of frequencies higher than about 30 MHz cannot penetrate the ionosphere

(4) Ground wave signals are more stable than the sky wave signals.

Sol. Above critical frequency (f_c), an electromagnetic wave penetrates the ionosphere and is not reflected by it.

25. A rectangular loop of wire, supporting a mass m, hangs with one end in a uniform magnetic field $\vec{B}$ pointing out of the plane of the paper. A clockwise current is set up such that i > mg/Ba, where a is the width of the loop. Then :



(1) The magnetic causes force with both a vertical and horizontal component along PQ, Since the displacement is vertical no work is current

(2) The magnetic field causes force with booth a vertical and horizontal component. Work has to be done by an external agent to maintain current

(3*) The weight rises due to a vertical force caused by the magnetic field but no work is done on the system

(4) The weight rises due to a vertical force caused by the magnetic field and work is extracted from the magnetic field

26. This question has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1 : Very large size telescopes are reflecting telescopes instead of refracting telescopes.

Statement 2 : It is easier to provide mechanical support to large size mirrors than large size lenses.

(1) Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation for statement-1

(2*) Statements-1 and statement-2 are true and Statement-2 is correct explanation for Statement-1

(3) Statement-1 is false and Statement-2 is true

(4) Statement-1 is true and Statement-2 is false

Sol. One side of mirror is opaque and another side is reflecting this is not in case of lens hence, it is easier to provide mechanical support to large size mirrors than large size lenses. Reflecting telescopes are based on the same principle except that the formation of images takes place by reflection instead of refraction.

27. A particle of charge 16×10^{-16} C moving with velocity 10 ms^{-1} along x-axis enters a region where magnetic field of induction $\vec{B}$ is along the y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative z-axis. If the charged particle continues moving along x-axis, the magnitude of $\vec{B}$ is :

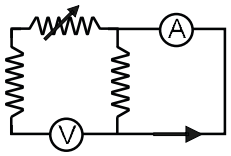
(1) 2×10^3 Wb m^{-2} (2) 16×10^3 Wb m^{-2} (3*) 1×10^3 Wb m^{-2} (4) 4×10^3 Wb m^{-2}

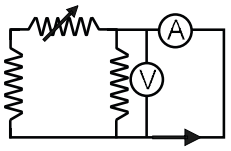
Sol. Since particle is moving undeflected

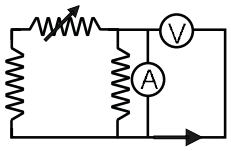
so, $q_E = qvB$

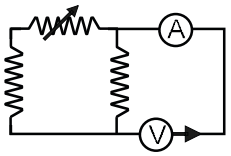
$$\Rightarrow B = \frac{E}{V} = \frac{10^4}{10} = 10^3 \text{ wb/ m}^2$$

28. Correct set up to verify Ohm's law is :

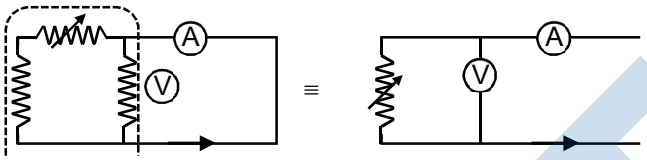
(1) 

(2*) 

(3) 

(4) 

Sol. In ohm's law, we check V = IR where I is the correct flowing through a resistor and V is the potential difference across that resistor. Only option (2) fits the above criteria. Remember that ammeter is connected in series with resistance and voltmeter parallel with the resistance.



29. A body of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be :

(1) 15.5 m (2) 12.5 m (3*) 17 m (4) 15 m

30. The ratio of the coefficient of volume expansion of a gas container to that of a viscous liquid kept inside the container is 1 : 4. What fraction of the inner volume of the container should the liquid occupy so that the volume of the remaining vacant space will be same at all temperatures ?

(1) 2 : 5 (2) 1 : 8 (3*) 1 : 4 (4) 1 ; 64

Sol. When there is no change in liquid level in vessel then $\gamma'_{real} = \gamma'_{vessel}$

Change in volume in liquid relative to vessel

$\Delta V_{app} = V\gamma'_{app}\,\Delta\theta = V(\gamma'_{real} - \gamma'_{vessel})$

PART-B-CHEMISTRY

31. Bond distance in HF is 9.17×10^{-11} m. Dipole moment of HF is 6.104×10^{-30} Cm. The percent ionic character in HF will be : (electron charge = 1.60×10^{-19} C)

(1) 61.0 % (2) 38.0% (3) 35.5 % (4*) 41.5 %

Sol. Given $e = 1.60 \times 10^{-19}$ C

$d = 9.17 \times 10^{-11}$ m

From $\mu = e \times d$

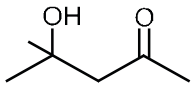
$\mu = 1.60 \times 10^{-19} \times 9.17 \times 10^{-11}$

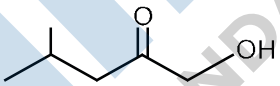
$= 14.672 \times 10^{-30}$

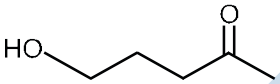
$$\% \text{ ionic character} = \frac{\text{Observed dipole moment}}{\text{Dipole moment for 100\% ionic bond}}$$

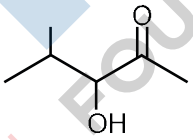
$$= \frac{6.104 \times 10^{-30}}{14.672 \times 10^{-30}} \times 100 = 41.5\%$$

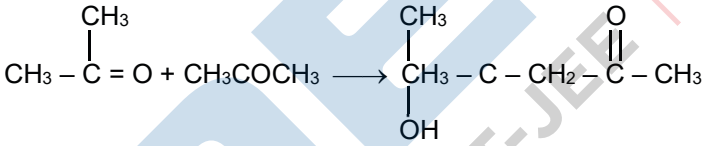
32. Which of the following is the product of aldol condensation ?

(1*) 

(2) 

(3) 

(4) 

Sol. 

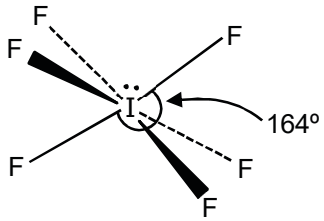
4–Hydroxy–4–methyl–2–pentanone

33. The shape of IF_6^- is :

(1) Pyramidal (2) Octahedral

(3*) Trigonally distorted octahedron (4) Square antiprism

Sol. The structure of IF_6^- is distorted octahedral. This is due to presence of a "weak" lone pair.



34. The de Broglie wavelength of a car of mass 1000 kg and velocity 36 km/hr is :

($h = 6.63 \times 10^{-34}$ Js)

(1) 6.626×10^{-30} m (2) 6.626×10^{-31} m (3*) 6.626×10^{-38} m (4) 6.626×10^{-34} m

Sol. $\lambda = \frac{h}{m\nu}$

$h = 6.6 \times 10^{-34}$ J-s

$m = 1000$ kg

$\nu = 36\,km/hr = \frac{36 \times 10^3}{60 \times 60} m/sec = 10\,m/sec$

$\therefore \quad \lambda = \frac{6.6 \times 10^{-34}}{10^3 \times 10} = 6.6 \times 10^{-38} m$

35. Monocarboxylic acids are functional isomers of :

(1*) Esters (2) Amines (3) Ethers (4) Alcohols

Sol. Mono-carboxylic acids are functional isomers of esters. e.g.,

$\underset{\text{Acetic acid}}{CH_3COOH}$ $\underset{\text{Methyl formate}}{HCOOCH_3}$

36. Number of atoms in the following samples of substances is the largest in :

(1) 48.0 g of magnesium (2) 127.0 g of iodine

(3*) 4.0 g of hydrogen (4) 71.0 g of chlorine

Sol. 4g of hydrogen = 4 mole of hydrogen = $4 \times 6.023 \times 10^{23}$ atoms

71.0 gm of chlorine $= \frac{71.0}{71.0} = 1$ moles of chlorine

$= 6.023 \times 10^{23}$ atoms

127 gm of iodine $= \frac{1}{2}$ mole of I_2

$= \frac{127}{254} = 6.023 \times 10^{23} \times \frac{1}{2}$ atoms

48.0 gm of magnesium

$= \frac{48.0}{24.0} = 2 \times 6.023 \times 10^{23}$ atoms

$\therefore$ 4.0 gm H_2 has largest number of atoms.

37. The correct order of viscosity of the following liquids will be :

(1*) dimethyl ether < methyl alcohol < water < glycerol

(2) methyl alcohol < glycerol < water < dimethyl ether

(3) glycerol < dimethyl ether < water < methyl alcohol

(4) Water < methyl alcohol < dimethyl ether < glycerol

Sol. The correct order of viscosity of the given liquids is

dimethyl ether < methyl alcohol < water < glycerol.

38. Which of the following polymer is a polyamide?

(1) Terylene (2) Rubber

(3*) Nylon (4) Vulcanised rubber

Sol. Nylon is a polyamide fibre. It is prepared by the condensation polymerization of adipic acid ($HOOC.(CH_2)_4COOH$) and hexamethylene diamine ($H_2N.(CH_2)_6.NH_2$).

39. What is the pH of a 10^{-4} M OH^- solution at 330K, if K_w at 330 K is $10^{-13.6}$?

(1) 9.0 (2) 10 (3*) 9.6 (4) 4

Sol. Given at 330 K

$K_w = 10^{-13.6}$

i.e. $pK_w = pH + pOH$

$\because$ $pOH = -\log [OH^-]$

$13.6 = pH + pOH$

$pOH = -\log 10^{-4}$

$13.6 = pH + 4$

$\therefore$ $pH = 13.6 - 4$

$= 9.6$

40. The numbers of protons, electrons and neutrons in a molecule of heavy water are respectively :

(1) 8, 10, 11 (2) 11, 10, 10 (3) 10, 11, 10 (4*) 10, 10, 10

Sol. Heavy water is D_2O hence number of electrons = 2 + 8 = 10

number of protons = 10

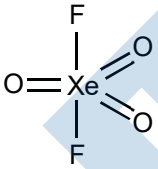
Atomic mass of D_2O = 4 + 16 = 20

hence number of neutron = Atomic mass – number of protons = 20 – 10 = 10

41. Which has trigonal bipyramidal shape ?

(1) $XeOF_2$ (2*) XeO_3F_2 (3) XeO_3 (4) $XeOF_4$

Sol. The shape of XeO_3F_2 is Trigonal bipyramidal.



(Trigonal bipyramidal Structure)

42. Natural glucose is termed D-glucoe because :

(1) –OH on the second carbon is on the right side in Fischer projection

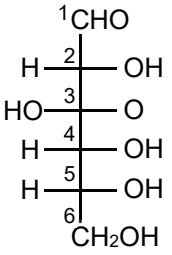
(2) It is dextrorotatory

(3*) –OH on the fifth carbon is on the right side in Fischer projection

(4) –OH on the sixth carbon is on the right side in Fischer projection

Sol. Fischer gave the prefix “D” to compounds whose bottom chiral has its OH to the right. So natural glucose is called D-glucose or dextrose.

Structure of D-Glucose :



D-Glucose

43. Smoke is an example of :

(1) Solid dispersed in solid (2*) Solid dispersed in gas

(3) Gas dispersed in liquid (4) Gas dispersed in solid

Sol. Smoke is an example of solid particles dispersed in gas.

44. When a small amount of $KMnO_4$ is added to concentrated H_2SO_4, a green oily compound is obtained which is highly explosive in nature. Compound may be :

(1) MnO_2 (2*) Mn_2O_7 (3) $MnSO_4$ (4) Mn_2O_3

Sol. $KMnO_4$ reacts with H_2SO_4 to form Mn_2O_7 which is highly explosive substance.

$2KMnO_4 + H_2SO_4 \longrightarrow K_2SO_4 + Mn_2O_7 + H_2O$

45. H_1–Receptor antagonists is a term associated with :

(1*) Antihistamins (2) Analgesics (3) Antacids (4) Antiseptics

Sol. The term “antihistamine” refers only to H_1 antagonists, which is also known as H_1-receptor antagonists and H_1-antihistamine.

46. The order of reactivity of the given haloalkanes towards nucleophile is :

(1) RBr > FI > RCl (2*) RI > RBr > RCl (3) RBr > RCl > RI (4) RCl > RBr > RI

Sol. For a given alkyl group, the order of reactivity is

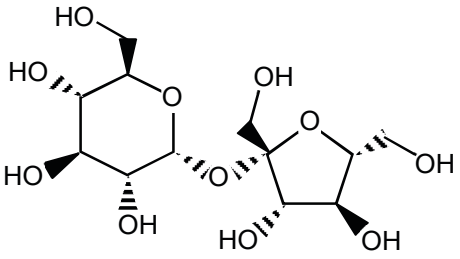
$$\underset{\text{Increasing bond energy}}{R-I > R-Br > R-Cl > R-F}$$

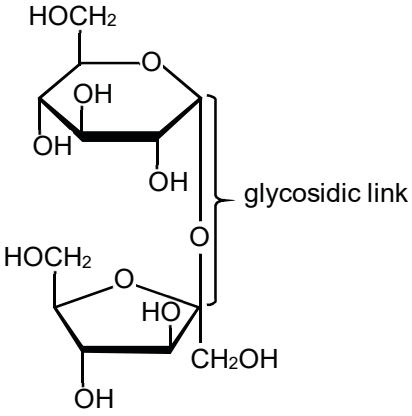
This order depends on the carbon-halogen bond energy; the carbon-fluorine bond energy is maximum and thus fluorides are least reactive while carbon-iodine bond energy is minimum hence iodides are most reactive.

47. Glycosidic linkage is a actually an :

(1) Carbonyl bond (2) Amide bond (3) Ester bond (4*) Ether bond

Sol. Glycosidic linkage is actually an ether bond as the linkage forming the rings in an oligosaccharide or polysaccharide is not just one bond, but the two bonds sharing an oxygen atom e.g. sucrose





48. In a face centered cubic lattice, atoms of A form the corner points and atoms of B form the face centered points. If two atoms of A are missing from the corner points, the formula of the ionic compounds is :

(1) AB_2 (2*) AB_4 (3) A_2B_5 (4) AB_3

Sol. A form corner points and two atoms of A are missing from corner

$\therefore$ Atoms at corner $(A) = 6 \times \frac{1}{8} = \frac{3}{4}$

Atoms at face centre $(B) = 6 \times \frac{1}{2} = 3$

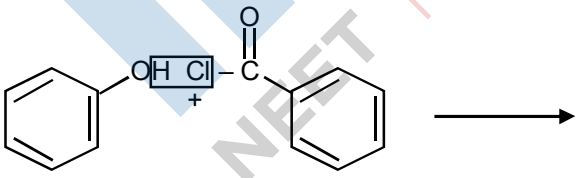
$\therefore$ $A_{3/4}\ B_3$ i.e., AB_4

49. The reaction of phenol with benzoyl chloride to give phenyl benzoate is known as :

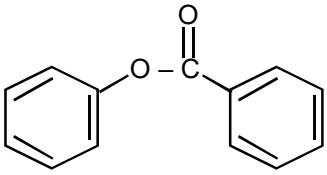
(1) Claisen reaction (2) Gatterman-Koch reaction

(3*) Schotten-Baumann reaction (4) Reimer-Tiemann reaction

Sol.



Phenol benzoyl chloride



Phenyl benzoate

"Schotten-Baumann reaction"

50. $N_2(g) + 3H_2(g) \rightleftharpoons 2\,NH_3(g),\ K_1$ (1)

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g),\ K_2$ (2)

$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g),\ K_3$ (3)

The equation for the equilibrium constant of the reaction $2\,NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$, (K_4) in terms of K_1, K_2 and K_3 is:

(1) $K_1 K_2 K_3$ (2) $\frac{K_1K_3^2}{K_2}$ (3*) $\frac{K_2K_3^3}{K_1}$ (4) $\frac{K_1K_2}{K_3}$

Sol. To calculate the value of K_4 in the given equation we should apply :

eqn. (2) + eqn. (3) × 3 – eqn. (1)

hence $K_4 = \frac{K_2K_3^3}{K_1}$

51. Identify incorrect statement :

(1) Copper (I) compound are diamagnetic

(2) Copper(I) compound are colouless except where colour results from change transfer

(3*) Cu_2O is colourless

(4) Cu_2S is black

Sol. Cu_2O is red in colour.

52. Equimolar solutions of the following compounds are prepared separately in water. Which will have the lowest pH value ?

(1) $CaCl_2$ (2*) $BeCl_2$ (3) $MgCl_2$ (4) $SrCl_2$

Sol. Metal halide on hydrolysis with water form corresponding hydroxides.

The basic strength of hydroxide increases as we move down in a group. This is because of the increase in size which results in decrease of ionization energy which weakens the strength of M – O bonds in MOH and thus increases the basic strength.

$\underset{\text{Amphoteric}}{Be(OH)_2}$ $\underset{\text{Weak base}}{Mg(OH)_2}$

$Ca(OH)_2\ Sr(OH)_2\ \underset{\text{Strong base}}{Ba(OH)_2}$

Hence, $Be(OH)_2$ will have lowest pH.

53. Given :

$$E^0_{\frac{1}{2}Cl_2/Cl^-} = 1.36V, E^0_{Cr^{3+}/Cr} = 0.74V,\ E^0_{Cr_2O_7^{2-}/Cl^{3+}} = 1.33V, E^0_{MnO_4^-/Mn^{2+}} = 1.51V$$

The correct order of reducing power of the species (Cr, Cr^{3+}, Mn^{2+} and Cl^-) will be:

(1) $Cr^{3+} < Cl^- < Cr < Mn^{2+}$ (2*) $Mn^{2+} < Cl^- < Cr^{3+} < Cr$

(3) $Mn^{2+} < Cr^{3+} < Cl^- < Cr$ (4) $Cr^{3+} < Cl^- < Mn^{2+} < Cr$

Sol. Lower the value of reduction potential higher will be reducing power hence the correct order will be

$Mn^{2+} < Cl^- < Cr^{3+} < Cr$

54. The order of basicity of amines in gaseous state is :

(1) $NH_3 > 1^o > 2^o > 3^o$ (2) $3^o > 2^o > NH_3 > 1^o$ (3) $1^o > 2^o > 3^o > NH_3$ (4*) $3^o > 2^\circ > 1^o > NH_3$

Sol. The correct order of relative basicity of amines in the gas phase is $3^o > 2^\circ > 1^o > NH_3$. The alkyl group releases electron and thus, tends to disperse the positive charge of the alkyl ammonium ion and therefore stabilizes it. Since, NH_4^+ (from NH_3) has no such alkyl group, it is not stabilized to such an extent as alkylammonium ion.

55. Which of the following statements/relationships is not correct in thermodynamic changes ?

(1) $w = -nRT\ln\frac{V_2}{V_1}$ (Isothermal reversible expansion of an ideal gas.)

(2) For a system at constant volume, heat involved merely changes of internal energy.

(3) $\Delta U = 0$ (Isothermal reversible expansion of a gas.)

(4*) $w = nRT\ln\frac{V_2}{V_1}$ (Isothermal reversible expansion of an ideal gas.)

Sol. For isothermal reversible expansion.

$$w = -nRT\ln\frac{V_2}{V_1}$$

56. Identify the incorrect statement :

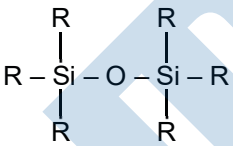
(1) $(Si_3O_9)^{6-}$ has cyclic structure

(2*) Trialkylchlorosilance on hydrolysis gives R_3SiOH

(3) In $(Si_3O_9)^{6-}$, tetrahedral SiO_4 units share two oxygen atoms

(4) $SiCl_4$ undergoes hydrolysis to give H_4SiO_4

Sol. The hydrolysis of Trialkylchlorosilane R_3SiCl yields dimer :



57. Which is the correct order of second ionization potential of C, N, O and F in the following ?

(1) C > N > O > F (2) F > O > N > C (3) O > N > F > C (4*) O > F > N > C

Sol. The second ionization potential means removal of electron from cation

$C^+ = 1s^2\ 2s^2\ 2p^1$, $N^+ = 1s^2\ 2s^2\ 2p^2$

$O^+ = 1s^2\ 2s^2\ 2p^3$, $F^+ = 1s^2\ 2s^2\ 2p^4$

Therefore O > F > N > C

58. The rate constant of a zero order reaction is 2.0×10^{-2} mol L^{-1} s^{-1}. If the concentration of the reactant after 25 seconds is 0.5 M. What is the initial concentration ?

(1) 12.5 M (2) 0.5 M (3*) 1.0 M (4) 1.25 M

Sol. For a zero order reaction

Rate constant $= k = \frac{a - x}{t}$

$$2 \times 10^{-2} = \frac{a - 0.5}{25}$$

$a - 0.5 = 0.5$

$a = 1.0$ M

59. Vapour pressure of pure benzene is 119 torr and that of toluene is 37.0 torr at the same temperature. Mole fraction of toluene in vapour phase which is in equilibrium with a solution of benzene and toluene having a mole fraction of toluene 0.50, will be :

(1) 0.137 (2) 0.205 (3) 0.435 (4*) 0.237

Sol. $P_A = P_A^0 \times x_A =$ total pressure × y_A

$P_B = P_B^0 \times x_B =$ total pressure × y_B

where x and y represents mole fraction in liquid and vapour phase respectively.

$$\frac{P_B^0 x_B}{P_A^0 x_A} = \frac{y_B}{y_A}; \frac{P_B^0 (1 - x_A)}{P_A^0 x_A} = \frac{1 - y_A}{y_A}$$

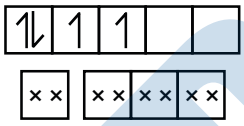
on putting values $\frac{119(1 - 0.50)}{37 \times 0.50} = \frac{1 - y_A}{y_A}$

on solving $y_A = 0.237$

60. The magnetic moment of the complex anion $[Cr(NO)(NH_3)(CN)_4]^{2-}$ is :

(1) 1.73 BM (2) 3.87 BM (3*) 2.82 BM (4) 5.91 BM

Sol. in $[Cr(NO)(NH_3)(CN)_4]^{2-}$, $Cr^{2+}(d^4)$ is given as :



i.e., 2 unpaired electrons

$\mu = \sqrt{2(2+2)} = \sqrt{8} = 2.82$

PART-C-MATHEMATICS

61. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P. is :

(1*) 13 m (2) 12 m (3) 15 m (4) 10 m

Sol. If d be the common difference, then

$m = a_4 - a_7 + a_{10} - a_7 + 3d = a_7$

$$S_{13} = \frac{13}{2}[a_1 + a_{13}] = \frac{13}{2}[a_1 + a_7 + 6d]$$

$$= \frac{13}{2}[2a_7] = 13a_7 = 13m$$

62. On the sides AB, BC, CA of a ΔABC, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are :

(1) 215 (2*) 205 (3) 210 (4) 220

Sol. Required number of triangles

$= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 205$

63. The point of intersection of the normals to the parabola $y^2 = 4x$ at the ends of its latus rectum is :

(1) (2, 0) (2*) (3, 0) (3), (0, 3) (4) (0, 2)

Sol. We know that point of intersection of the normal to the parabola $y^2 = 4ax$ at the ends of its latus rectum is (3a, 0)

Hence required point of intersection = (3, 0)

64. Let $a = \text{Im.}\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. The set A = { a : | z | = 1 and $z \neq \pm 1$} is equal to :

(1*) (– 1, 1) (2) [0, 1) (3) [– 1, 1] (4) (– 1, 0]

Sol. Let $z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$

Now, $$\frac{1+z^2}{2iz} = \frac{1+x^2-y^2+2ixy}{2i(x+iy)} = \frac{(x^2-y^2+1)+2ixy}{2ix-2y}$$

$$= \frac{(x^2-y^2+1)+2ixy}{-2y+2ix} \times \frac{-2-2ix}{-2y-2ix}$$

$$= \frac{y(x^2+y^2-1)+x(x^2+y^2+1)i}{2(x^2+y^2)}$$

$$a = \frac{x(x^2+y^2+1)}{2(x^2+y^2)}$$

Since, $|x| = 1 \Rightarrow \sqrt{x^2+y^2} = 1$

$\Rightarrow$ $x^2 + y^2 = 1$

$\therefore \quad a = \dfrac{x(x^2+y^2+1)}{2(x^2+y^2)}$

Also $z \neq 1 \Rightarrow x + iy \neq 1$

$\therefore \quad A = (-1, 1)$

65. If two lines L_1 and L_2 in space, are defined by

$L_1 = \left\{ x = \sqrt{\lambda}\, y + (\sqrt{\lambda} - 1),\ z = (\sqrt{\lambda} - 1)\, y + \sqrt{\lambda} \right\}$ and

$L_2 = \left\{ x = \sqrt{\mu}\, y + (1 - \sqrt{\mu}),\ z = (1 - \sqrt{\mu})\, y + \sqrt{\mu} \right\}$

then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that :

(1*) $\lambda = \mu$ (2) $\lambda + \mu = 0$ (3) $\lambda \neq \mu$ (4) $\sqrt{\lambda} + \sqrt{\mu} = 1$

Sol. For L_1,

$x = \sqrt{\lambda} y + \left(\sqrt{\lambda} - 1\right) \Rightarrow y = \dfrac{x - \left(\sqrt{\lambda} - 1\right)}{\sqrt{\lambda}}$ ....(i)

$z = \left(\sqrt{\lambda} - 1\right) y + \sqrt{\lambda} \Rightarrow y = \dfrac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$ ....(ii)

From (i) and (ii)

$\dfrac{x - \left(\sqrt{\lambda} - 1\right)}{\sqrt{\lambda}} = \dfrac{y - 0}{1} = \dfrac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$ ....(A)

The equation (A) is the equation of line L_1.

Similarly equation of line L_2 is

$\dfrac{x - \left(1 - \sqrt{\mu}\right)}{\sqrt{\mu}} = \dfrac{y - 0}{1} = \dfrac{z - \sqrt{\mu}}{1 - \sqrt{\mu}}$ ....(B)

Since $L_1 \perp L_2$, therefore

$\sqrt{\lambda}\sqrt{\mu} + 1 \times 1 + \left(\sqrt{\lambda} - 1\right)\left(1 - \sqrt{\mu}\right) = 0$

$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0 \Rightarrow \sqrt{\lambda} = -\sqrt{\mu}$

$\Rightarrow \lambda = \mu$

66. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then the value of α for which the vectors $\vec{\mu} = (\alpha - 2)\vec{a} + \vec{b}$ and $\vec{v} = (2 + 3\alpha)\vec{a} - 3\vec{b}$ are collinear is :

(1) $\dfrac{3}{2}$ (2*) $\dfrac{2}{3}$ (3) $\dfrac{-2}{3}$ (4) $\dfrac{-3}{2}$

Sol. Since, $\vec{u}$ and $\vec{v}$ are collinear, therefore $k\vec{u} + \vec{v} = 0$

$\Rightarrow [k(\alpha - 2) + 2 + 3\alpha]\vec{a} + (k - 3)\vec{b} = 0$ ...(i)

Since $\vec{a}$ and $\vec{b}$ are non-collinear, then for some constant m and n,

$m\vec{a}+n\vec{b}=0 \Rightarrow m=0,n=0$

Hence from equation (i)

$k - 3 = 0 \Rightarrow k = 3$

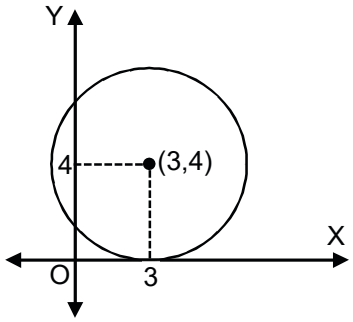
And $k(\alpha - 2) + 2 + 3\alpha = 0$

$\Rightarrow 3(\alpha-2)+2+3\alpha=0 \Rightarrow \alpha=\frac{2}{3}$

67. If the circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x, then a equals :

(1) ± 3 (2*) ± 4 (3) ± 2 (4) 0

Sol.



$x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$

Radius $= 4 = \sqrt{9+16+(25-a^2)}$

$\Rightarrow a = \pm 4$

68. The sum of the series ; $(2)^2 + 2(4)^2 + 3(6)^2 + \ldots\ldots$ upto 10 terms is :

(1) 11300 (2*) 12100 (3) 12300 (4) 11200

Sol. $2^2 + 2(4)^2 + 3(6)^2 + \ldots..$ upto 10 terms

$= 2^2 [1^3 + 2^3 + 3^3 + \ldots..$ upto 10 terms]

$=4\left(\frac{10\times 11}{2}\right)^2 = 12100$

69. **Statement-1:** The system of linear equations

$x + (\sin\alpha)\, y + (\cos\alpha)\, z = 0$

$x + (\cos\alpha)\, y + (\sin\alpha)\, z = 0$

$x - (\sin\alpha)\, y - (\cos\alpha)\, z = 0$

has a non-trivial solution for only one value of α lying in the interval $\left(0, \frac{\pi}{2}\right)$.

Statement-2: The equation in α $\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$ has only one solution lying in the interval $\left(0, \frac{\pi}{2}\right)$.

(1) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(2*) Statement-1 is true; Statement-2 is false.

(3) Statement-1 is false; Statement-2 is true.

(4) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

Sol. $\Delta_1\alpha = \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & -\cos\alpha \end{vmatrix} = \begin{vmatrix} 0 & \sin\alpha-\cos\alpha & \cos\alpha-\sin\alpha \\ 0 & \cos\alpha+\sin\alpha & \sin\alpha-\cos\alpha \\ 1 & -\sin\alpha & -\cos\alpha \end{vmatrix}$

$= (\sin\alpha - \cos\alpha)^2 - (\cos^2\alpha - \sin^2\alpha)$

$= \sin^2\alpha + \cos^2\alpha - 2\sin\alpha \cdot \cos\alpha - \cos^2\alpha + \sin^2\alpha$

$= 2\sin^2\alpha - 2\sin\alpha \cdot \cos\alpha$

$= 2\sin\alpha(\sin\alpha - \cos\alpha)$

Now, $\sin\alpha - \sin\alpha = 0$ for only

$\alpha = \frac{\pi}{4} \text{in} \left(0, \frac{\pi}{2}\right)$

$\therefore\ \Delta_1 = 2(\sin\alpha) \times 0 = 0,$

since value of $\sin\alpha$ is finite for $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence non-trivivial solution for only one value of α in $\left(0, \frac{\pi}{2}\right)$

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ 2\cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$\Rightarrow 2\cos\alpha(\sin^2\alpha - \cos^2\alpha) = 0$

$\therefore \cos\alpha = 0$ or $\sin^2\alpha - \cos^2\alpha = 0$

But $\cos\alpha = 0$ not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

Hence, there is no solution.

70. The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$, satisfies :

(1*) $\alpha^2 + 3\alpha - 4 = 0$ (2) $\alpha^2 - 7\alpha + 6 = 0$ (3) $\alpha^2 - 5\alpha + 4 = 0$ (4) $\alpha^2 + 5\alpha - 6 = 0$

Sol. $\frac{x-5}{x^2+5x-14} > 0 \Rightarrow x^2 + 5x - 14 < x - 5$

$\Rightarrow x^2 + 4x - 9 < 0$

$\Rightarrow \alpha = -5, -4, -3, -2, -1, 0, 1$

$\alpha = -5$ does not satisfy any of the options

$\alpha = -4$ satisfy the option (a) $\alpha^2 + 3\alpha - 4 = 0$

71. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx$ is :

(1) 4π (2) $\frac{\pi}{2}$ (3) π (4*) $\frac{\pi}{4}$

Sol. $I = \int_{\pi/2}^{\pi/2} \frac{\sin^2}{1+2x^2}dx$ ....(i)

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2}{1+2^{-x}}dx$, by replacing x by

$\left(\frac{\pi}{2}-\frac{\pi}{2}-x\right)$

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \sin^2 x dx = \frac{1}{2}\int_{-\pi/2}^{\pi/2}(1-\cos 2x)dx$

$\Rightarrow I = \frac{1}{4}\left[x+\frac{\sin 2x}{2}\right]_{-\pi/2}^{\pi/2}$

$= \frac{1}{4}\left[\left(\frac{\pi}{2}+\frac{\sin\pi}{2}\right)-\left(-\frac{\pi}{2}+\frac{\sin(-\pi)}{2}\right)\right]$

$\Rightarrow I = \frac{1}{4}\left[\frac{\pi}{2}+\frac{\pi}{2}\right] = \frac{\pi}{4}$

72. Let f be a composite function of x defined by $f(u) = \frac{1}{u^2+u-2}, u(x) = \frac{1}{x-1}$. Then the number of points x where f is discontinuous is :

(1) 2 (2) 1 (3*) 3 (4) 4

Sol. $\mu(x) = \frac{1}{x-1}$, which is discontinuous at x = 1

$f(u) = \frac{1}{u^2+u-2} = \frac{1}{(u+2)(u-1)}$, which is discontinuous at u = –2, 1

when u = –2, then $\frac{1}{x-1} = -2 \Rightarrow x = \frac{1}{2}$

when u = 1, then $\frac{1}{x-1} = 1 \Rightarrow x = 2$

Hence given composite function is discontinuous at three points $x = 1, \frac{1}{2}$ and 2.

73. If the median and the range of four numbers {x, y, 2x +y, x – y}, where 0 < y < x < 2y are 10 and 28 respectively, then the mean of the numbers is :

(1) 5 (2*) 14 (3) 10 (4) 18

Sol. Since 0 < y < x < 2y

$\therefore y > \frac{x}{2} \Rightarrow x - y < \frac{x}{2}$

$\therefore x - y < y < x < 2x + y$

Hence median $= \frac{y + x}{2} = 10$

$\Rightarrow x + y = 20$ ....(i)

And range = (2x + y) – (x – y) = x + 2y

But range = 28

$\therefore$ x + 2y = 28 ....(ii)

From equation (i) and (ii),

x = 12, y = 8

$\therefore$ Mean

$= \frac{(x - y) + y + x + (2x + y)}{4} = \frac{4x + y}{4}$

$= x + \frac{y}{4} = 12 + \frac{8}{4} = 14$

74. A, B, C try to hit a target simultaneously but independently. Their respective probabilities of hitting the targets are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is :

(1) $\frac{9}{64}$ (2) $\frac{7}{8}$ (3*) $\frac{21}{64}$ (4) $\frac{7}{32}$

Sol. P(A or B but not by C) $= P((A \cup B) \cap \bar{C})$

$= P(A \cup B) \times P(\bar{C})$

$= [P(A) + P(B) - P(A \cap B)] \times P(\bar{C})$

$= \left[\frac{3}{4} + \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}\right] \times \frac{3}{8} = \left(\frac{6 + 4 - 3}{8}\right) \times \frac{3}{8} = \frac{21}{24}$

75. If $S = \tan^{-1}\left(\frac{1}{n^2 + n + 1}\right) + \tan^{-1}\left(\frac{1}{n^2 + 3n + 3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1 + (n + 19)(n + 20)}\right)$, then tan S is equal to :

(1*) $\frac{20}{n^2 + 20n + 1}$ (2) $\frac{n}{401 + 20n}$ (3) $\frac{n}{n^2 + 20n + 1}$ (4) $\frac{20}{401 + 20n}$

Sol. We know that,

$$\tan^{-1}\frac{1}{1+2}+\tan^{-1}\frac{1}{1+2\times3}+\tan^{-1}\frac{1}{1+3\times4}+....+\tan^{-1}\frac{1}{1+(n+1)n}+\tan^{-1}\frac{1}{1+n(n+1)}$$

$$+.....+\frac{1}{1+(n+19)(n+20)}=\tan^{-1}\frac{n+19}{n+21}$$

$$\tan^{-1}\frac{1}{1+n(n+1)}+\tan^{-1}\frac{1}{1+(n+1)(n+2)}+....+\frac{1}{1+(n+19)(n+20)}=\tan^{-1}\frac{n+19}{n+21}-\tan^{-1}\frac{n-1}{n+1}$$

$$\tan^{-1}\left(\frac{1}{n^2+n+1}\right)+\tan^{-1}\left(\frac{1}{n^2+3n+3}\right)+....+\tan^{-1}\frac{1}{1+(n+19)(n+20)}$$

$$=\tan^{-1}\left(\frac{\frac{n+19}{n+21}-\frac{n-1}{n+1}}{1+\frac{n+19}{n+21}\times\frac{n-1}{n+1}}\right)$$

$$=\tan^{-1}\frac{20}{n^2+20n+1}=S$$

$$\therefore\ \tan^{-1}S=\frac{20}{n^2+20n+1}$$

76. The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is :

(1) 4 (2) 6 (3) 5 (4*) 3

Sol. $\sin 2x - 2\cos x + 4\sin x = 4$

$\Rightarrow 2\sin x\,.\cos x - 2\cos x + 4\sin x - 4 = 0$

$\Rightarrow (\sin x - 1)(\cos x - 2) = 0$

$\because\ \cos x - 2 \neq 0,\ \therefore\ \sin x = 1$

$\therefore\ x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

77. The integral $\int\frac{x}{2-x^2+\sqrt{2-x^2}}dx$ equals :

(1) $-x\log\left|1-\sqrt{2-x^2}\right|+C$ (2) $\log\left|1+\sqrt{2+x^2}\right|+C$

(3*) $-\log\left|1+\sqrt{2-x^2}\right|+C$ (4) $x\log\left|1-\sqrt{2+x^2}\right|+C$

Sol. $I=\int\frac{xdx}{2-x^2+\sqrt{2-x^2}}$

Put $t=\sqrt{2-x^2}, \frac{dt}{dx}=\frac{1}{2\sqrt{2-x^2}}.(-2x)$

$\Rightarrow -t\,dt = x\,dx$

$$I=\int\frac{(-t)dt}{t^2+t}=\int\frac{1}{t+1}dt=-\log|t+1|$$

$$-\log\left|\sqrt{2-x^2}+1\right|+C$$

78. If the projections of a line segment on the x, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the line segment is :

(1) 6 (2) 12 (3) 9 (4*) 7

Sol. Length of the line segment

$= \sqrt{(2)^2 + (3)^2 + (6)^2} = 7$

79. If the curves $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ intersect at right angles, then a value of α is :

(1) $\frac{3}{4}$ (2) 2 (3*) $\frac{4}{3}$ (4) $\frac{1}{2}$

Sol. $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1 \Rightarrow \frac{2x}{\alpha} + \frac{2y}{4} \cdot \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-4x}{\alpha y}$ ....(i)

$y^3 = 16x \Rightarrow 3y^2 \cdot \frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{16}{3y^2}$ ....(ii)

Since curves intersects at right angles

$\therefore \frac{-4x}{\alpha y} + \frac{16}{3y^2} = -1 \Rightarrow 3\alpha y^3 = 64x$

$\Rightarrow \alpha = \frac{64x}{3 \times 16x} = \frac{4}{3}$

80. For integers m and n, both greater than 1, consider the following three statements.

P : m divides n

Q : m divides n^2

R : m is prime,

then

(1) $Q \rightarrow R$ (2) $Q \rightarrow P$ (3) $P \wedge Q \rightarrow R$ (4*) $Q \wedge R \rightarrow P$

Sol. (b) $\frac{8}{2} = 2, \frac{64}{4} = 16;$ but 4 is not prime.

Hence $P \wedge Q \rightarrow R$, false

(c) d $\frac{(6)^2}{12} = \frac{36}{12} = 3;$ but 12 is not prime

Hence $Q \rightarrow R$, false

(d) $\frac{(4)^2}{8} = \frac{16}{8} = 2; \frac{4}{8}$ is not an integer

Hence $Q \rightarrow P$, false

81. Let A, other than I or – I, be a 2 × 2 real matrix such that $A^2 = I$, I being the unit matrix. Let Tr(A) be the sum of diagonal elements of A.

Statement-1: Tr(A) = 0

Statement-2: det.(A) = – 1

(1) Statement-1 is true; Statement-2 is false.

(2) Statement-1 is false; Statement-2 is true.

(3*) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(4) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

Sol. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$b(a + d) = 0$, $b = 0$ or $a = -d$ ....(1)

$c(a + d) = 0$, $c = 0$ or $a = -d$ ....(2)

$a^2 + bc = 1$, $bc + a^2 = 1$ ....(3)

'a' and 'd' are diagonal elements $a + d = 0$

statement-1 is correct.

Now, $\det(A) = ad - bc$

Now, from (3) $a^2 + bc = 1$ and $d^2 + bc = 1$

So, $a^2 - d^2 = 0$

Adding $a^2 + d^2 + 2bc = 2$

$= (a + d)^2 - 2ad + 2bc = 2$

or $0 - 2(ad - bc) = 2$

So, $ad - bc = 1 \Rightarrow \det(A) - 1$

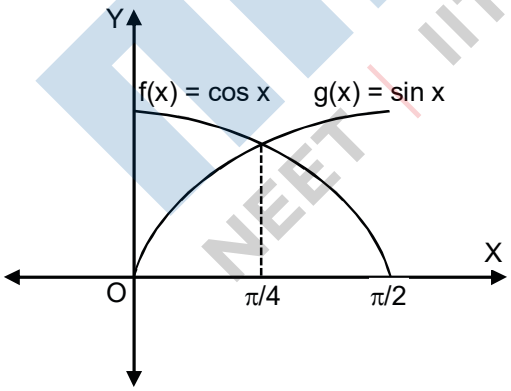
So, statement – 2 is also true.

But statement – 2 is not the correct explanation of statement-1.

82. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$ and above x-axis is :

(1) $2\sqrt{2}$ (2*) $2\sqrt{2} - 2$ (3) 0 (4) $2\sqrt{2} + 2$

Sol. $y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4} (\cos x - \sin x)\,dx = 2\left[\sin x + \cos x\right]_0^{\pi/4}$

$= 2\left[\frac{2}{\sqrt{2}} - 1\right] = \left(2\sqrt{2} - 2\right)$ sq. units

83. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers.

Statement-1: If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.

Statement-2: $\theta_1 = \theta_2$ for all c_2 and c_3.

(1*) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(2) Statement-1 is false; Statement-2 is true.

(3) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(4) Statement-1 is true; Statement-2 is false.

Sol. Two lines $-x + 5y + c_2 = 0$ and $-x + 5y + c_3 = 0$ are parallel to each other. Hence statement-1 is true, statement-2 is true and statement-2 is the correct explanation of statement-1.

84. The cost of running a bus from A to B, is Rs. $\left(av + \frac{b}{v}\right)$, where v km/h is the average speed of the bus.

When the bus travels at 30 km/h, the cost comes out to be Rs. 75 while at 40 km/h, it is Rs. 65. Then the most economical speed (in km/h) of the bus is :

(1*) 60 (2) 40 (3) 45 (4) 50

Sol. Let cost $C = av + \frac{b}{v}$

According to given question,

$30a + \frac{b}{30} = 75$ ....(i)

$40a + \frac{b}{40} = 65$ ....(ii)

On solving (i) and (ii), we get

$a = \frac{1}{2}$ and $b = 1800$

Now, $C = av + \frac{b}{v}$

$\Rightarrow \frac{dC}{dv} = a - \frac{b}{v^2}$

$\frac{dC}{dv} = 0 \Rightarrow a - \frac{b}{v^2} = 0$

$\Rightarrow v = \sqrt{\frac{b}{a}} = \sqrt{3600}$

$\Rightarrow v = 60$ kmph

85. If $f(x) = \sin(\sin x)$ and $f''(x) + \tan x\, f'(x) + g(x) = 0$, then $g(x)$ is :

(1) $\cos^2 x \cos(\sin x)$ (2) $\sin^2 x \sin(\cos x)$ (3) $\sin^2 x \cos(\cos x)$ (4*) $\cos^2 x \sin(\sin x)$

Sol. $f(x) = \sin(\sin x)$

$\Rightarrow$ $f'(x) = \cos(\sin x) \cdot \cos x$

$\Rightarrow$ $f''(x) = -\sin(\sin x) \cdot \cos^2 x + \cos(\sin x) \cdot (-\sin x)$

$= -\sin^2 x \cdot \sin(\sin x) - \sin x \cdot \cos(\sin x)$

Now, $f''(x) + \tan \cdot f'(x)\, g(x) = 0$

$\Rightarrow$ $g(x) = \cos^2 x \cdot \sin(\sin x) + \sin x \cdot \cos(\sin x) - \sin x \cdot \cos x \cdot \cos(\sin x)$

$\Rightarrow$ $g(x) = \cos^2 x \cdot \sin(\sin x)$

86. The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}} + 3^{\frac{1}{5}}\right)^{10}$ is :

(1) 32 (2) 9 (3*) 41 (4) 25

Sol. $(2^{1/2} + 3^{1/5})^{10} = {}^{10}C_0(2^{1/2})^{10} + {}^{10}C_1(2^{1/2})^9(3^{1/5}) + \ldots\ldots + {}^{10}C_{10}(3^{1/5})^{10}$

Therefore are only two rational terms – first term and last term.

Now sum of two rational terms = $(2)^5 + (3)^2 = 32 + 9 = 41$

87. Let $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$, where N is the set of all natural numbers. Then the relation R is :

(1*) reflexive and transitive

(2) symmetric and transitive

(3) reflexive but neither symmetric nor transitive

(4) reflexive and symmetric

Sol. $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$

Now, $x^2 - 4xy + 3y^2 = 0$

$\Rightarrow (x - y)(x - 3y) = 0$

$\therefore x = y$ or $x = 3y$

$\therefore$ $R = \{(1, 1), (3, 1), (2, 2), (6, 2), (3, 3), (9, 3), \ldots\ldots\}$

Since (1, 1), (2, 2), (3, 3), ……. are present in the relation, therefore R is reflexive.

Since (3, 1) is an element of R but (1, 3) is not the element of R, therefore R is not symmetric

Here $(3, 1) \in R$ and $(1, 1) \in R \Rightarrow (3, 1) \in R$ $(6, 2) \in R$ and $(2, 2) \in R \Rightarrow (6, 2) \in R$

For all such $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$

Hence R is transitive.

88. A tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ meets x-axis at P and y-axis at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is the origin). Then R lies on :

(1*) $\frac{4}{x^2} - \frac{2}{y^2} = 1$ (2) $\frac{2}{x^2} - \frac{4}{y^2} = 1$ (3) $\frac{4}{x^2} + \frac{2}{y^2} = 1$ (4) $\frac{2}{x^2} + \frac{4}{y^2} = 1$

Sol. Equation of the tangent at the point 'θ' is $\frac{x\sec\theta}{} - \frac{y\tan\theta}{} = 1$

$\Rightarrow$ P = (a cos θ, 0) and Q = (0, –b cot θ)

Let R be (h, k) $\Rightarrow$ h = a cos θ, k = –b cot θ

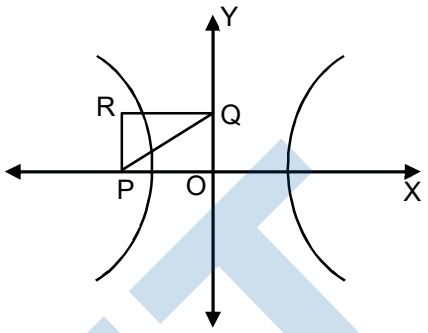
$$\Rightarrow \quad \frac{k}{h}=\frac{-b}{a\sin\theta} \Rightarrow \sin\theta=\frac{-bh}{ak} \quad \text{and} \quad \cos\theta=\frac{h}{a}$$

By squaring and adding,

$$\frac{b^2h^2}{a^2k^2}+\frac{h^2}{a^2}=1$$

$$\Rightarrow \frac{b^2}{k^2}+1\frac{a^2}{h^2}$$

$$\Rightarrow \frac{a^2}{h^2}-\frac{b^2}{k^2}=1$$



Now, given equation of hyperbola is $\frac{x^2}{4}-\frac{y^2}{2}=1$

$\Rightarrow a^2 = 4, b^2 = 2$

$\therefore$ R lies on $\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$ i.e., $\frac{4}{x^2}-\frac{2}{y^2}=1$

89. If the extremities of the base of an isosceles triangle are the points (2a, 0) and (0, a) and the equation of one of the sides is x = 2a, then the area of the triangle, in square units, is :

(1) $\frac{25a^2}{4}$ (2) $\frac{5}{4}a^2$ (3*) $\frac{5}{2}a^2$ (4) $5a^2$

Sol. Let y-coordinate of C = b

$\therefore$ C = (2a, b)

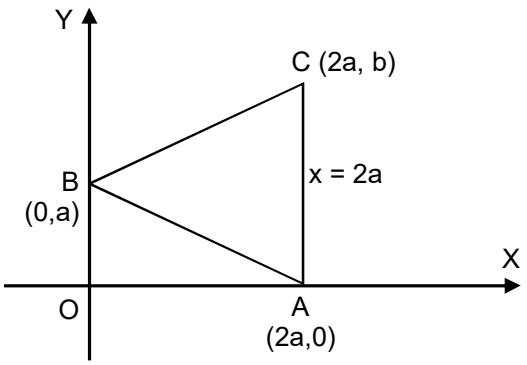
$$AB=\sqrt{4a^2+a^2}=\sqrt{5}a$$

Now, $AC = BC \Rightarrow b=\sqrt{4a^2+(b-a)^2}$

$\Rightarrow b^2 = 4a^2 + b^2 + a^2 – 2ab$

$$\Rightarrow 2ab=5a^2 \Rightarrow b=\frac{5a}{2}$$

$$\therefore C=\left(2a,\frac{5a}{2}\right)$$



Hence area of the triangle

$$=\frac{1}{2}\begin{vmatrix}2a & 0 & 1\\ 0 & a & 1\\ 2a & \frac{5a}{2} & 1\end{vmatrix}=\frac{1}{2}\begin{vmatrix}2a & 0 & 1\\ 0 & a & 1\\ 0 & \frac{5a}{2} & 0\end{vmatrix}$$

$$=\frac{1}{2}\times 2a\left(-\frac{5a}{2}\right)=-\frac{5a^2}{2}$$

Since area is always +ve, hence area $= \frac{5a^2}{2}$ sq. unit

90. If a curve passes through the point $\left(2, \frac{7}{2}\right)$ and has slope $\left(1-\frac{1}{x^2}\right)$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is – 2 is :

(1*) $\frac{-3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{3}{2}$ (4) $\frac{-5}{2}$

Sol. Slope $= \frac{dy}{dx} = 1-\frac{1}{x^2}$

$\Rightarrow \int dy = \int\left(1-\frac{1}{x^2}\right)dx$

$\Rightarrow y = x+\frac{1}{x}+C$, which is the equation of the curve

since curve passes through the point $\left(2,\frac{7}{2}\right)$

$\therefore \frac{7}{2} = 2+\frac{1}{2}+C \Rightarrow C = 1$

$\therefore y = x+\frac{1}{x}+1$

when x = –2, then $y = -2+\frac{1}{-2}+1 = \frac{-3}{2}$